# Meanings as proposals: An algebraic inquisitive semantics

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## Goal and approach

We aim for a semantics that unlike classical and basic inquisitive semantics [4] distinguishes:

(1) $p \vee q$
(2) $p \vee q \vee (p \wedge q)$

Starting from a particular view on meaning, we derive such a semantics guided by general algebraic concerns.

## Relevance

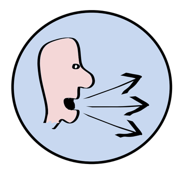
- Enables a Gricean explanation of why (1) but not (2) pragmatically implies $\neg(p \wedge q)$.
- Relates to algebraically similar formalisms, e.g., propositional dynamic logic.
- As in [2], the semantics can model epistemic modal 'might $p$' as $p \vee \top$.

## Main results

- A generalized version of unrestricted inquisitive semantics [2], though with new notions of entailment and implication.
- A deeper conceptual understanding of it, as a semantics of proposals.
- An algebraic characterisation as a dioid.

## Meanings as proposals

For $\mathbf{W}$ a set of worlds, a meaning $A$ is a set of update functions on epistemic states, $A \subseteq \wp\mathbf{W}^{\wp\mathbf{W}}$. We think of such sets as follows:



$A$ represents a proposal to update the common ground with any $f \in A$.

We derive a semantics from this view.

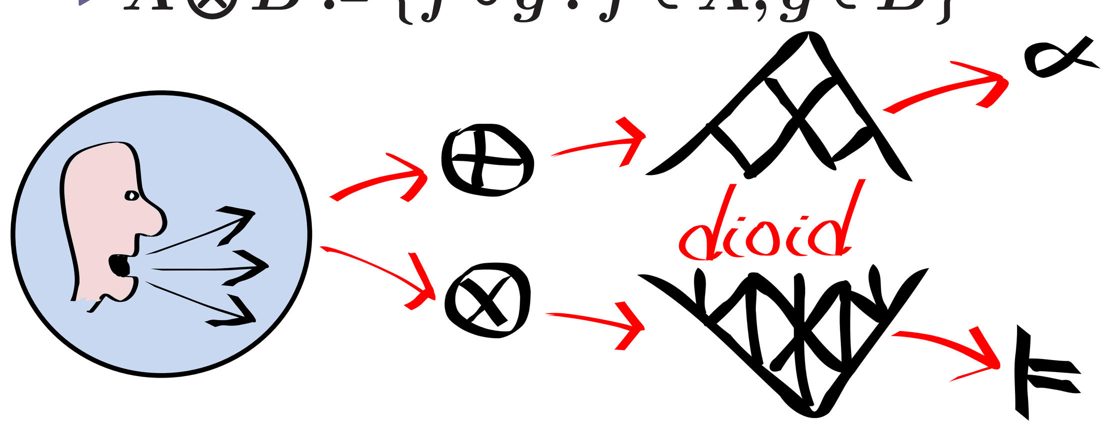
## An algebra of proposals

To let (1) $\not\equiv$ (2), either the laws of absorption must fail, or disjunction must lack idempotence. Without absorption, the algebraic structure would not be a lattice (cf. right column) but, at best, two semi-lattices. Hence, to be safe, we choose two conceptual footholds, one for each half. We motivate definitions for $\oplus$ and $\otimes$ by spelling out the 'proposal'-view:

(3) 'Let's do an $f \in A$ or a $g \in B$' ≡ 'Let's do an $f \in A \cup B$'
(4) 'Let's do an $f \in A$ and a $g \in B$' ≡ 'Let's do a $f \circ g$, $f \in A$, $g \in B$'

Here $f \circ g$ is function composition. Thus:

- $A \oplus B := A \cup B$
- $A \otimes B := \{f \circ g : f \in A, g \in B\}$




This gives us the following properties:

- $\langle \wp(\wp\mathbf{W}^{\wp\mathbf{W}}), \oplus, \varnothing \rangle$ is a join-semilattice
- $\langle \wp(\wp\mathbf{W}^{\wp\mathbf{W}}), \otimes, \{\lambda x. x \cap \mathbf{W}\} \rangle$ is a monoid
- No absorption ← good news!
- $\otimes$ distributes over $\oplus$
- $A \oplus \varnothing = A$, $A \otimes \varnothing = \varnothing$

And hence:

- $\langle \wp(\wp\mathbf{W}^{\wp\mathbf{W}}), \oplus, \varnothing, \otimes, \{\lambda x. x \cap \mathbf{W}\} \rangle$ is a dioid, i.e., an idempotent semiring.

Each operation yields a natural order:

- $A \vDash B :\Longleftrightarrow \exists C. B \otimes C = A$
- $A \propto B :\Longleftrightarrow \exists C. B \oplus C = A$

For entailment, we have $A \otimes B \vDash A$, but not $A \vDash A \oplus B$, in accordance with:

(5) 'Let's have tea and cake' implies 'Let's have tea'
(6) 'Let's have tea' doesn't imply 'Let's have tea or cake'

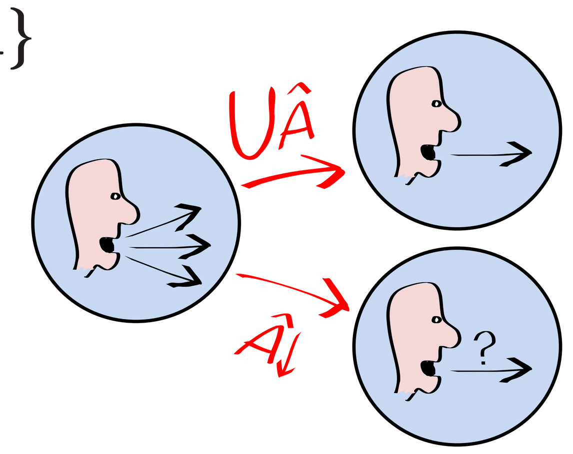
The other order ($\propto$) captures a basic notion of compliance (cf. [3]). We have $A \oplus B \propto A$, but not $A \propto A \otimes B$.

## Static proposals

If a function is eliminative and distributive, i.e., it only provides information about the world, we call it (and proposals containing only such functions) 'static'. A static function $f$ can be represented by an object $f(\mathbf{W}) \subseteq \mathbf{W}$ [1], hence a static proposal $A$ by an object $\widehat{A} \subseteq \wp\mathbf{W}$:

- $\widehat{A} := \{f(\mathbf{W}) : f \in A\}$

A static proposal $A$ yields the information $\bigcup \widehat{A}$ and the request $\widehat{A}{\downarrow}$ (cf. right column; $\downarrow$ is downward closure).




## Proposal implication

Dioids lack a natural complement operation. We suggest that natural language implication (and negation) operates on requests, not on proposals. We define proposal implication $\Rightarrow$, in terms of the relative pseudo-complement $\oslash$ of basic inquisitive semantics [4]:

- $A \Rightarrow B = \{\lambda x. x \cap \alpha : \alpha \in \mathsf{MAX}(\widehat{A}{\downarrow} \oslash \widehat{B}{\downarrow})\}$

I.e., $A \Rightarrow B$ is the proposal to update in a way that resolves exactly the request $\widehat{A}{\downarrow} \oslash \widehat{B}{\downarrow}$.

## Unrestricted inquisitive semantics

We define unrestricted inquisitive semantics for propositional logic, with $\neg\varphi := \varphi \to \bot$:

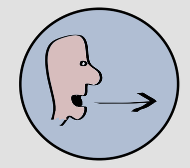
- $[p] = \{\lambda x. x \cap \{w : w(p) = 1\}\}$;
- $[\top] = \{\lambda x. x \cap \alpha : \alpha \subseteq \mathbf{W}\}$, $[\bot] = \{\lambda x. x \cap \varnothing\}$
- $[\varphi \vee \psi] = [\varphi] \oplus [\psi]$
- $[\varphi \wedge \psi] = [\varphi] \otimes [\psi]$
- $[\varphi \to \psi] = [\varphi] \Rightarrow [\psi]$

With the following conservativity results:

- For all $\varphi$, $\bigcup \widehat{[\varphi]} = [\varphi]_{\text{classical sem.}}$
- For all $\varphi$, $\widehat{[\varphi]}{\downarrow} = [\varphi]_{\text{basic-inquisitive-sem.}}$

## Comparison: meanings as information

For $\mathbf{W}$ a set of worlds, $w_0$ the actual world, we can think of a set $A \subseteq \mathbf{W}$ as:

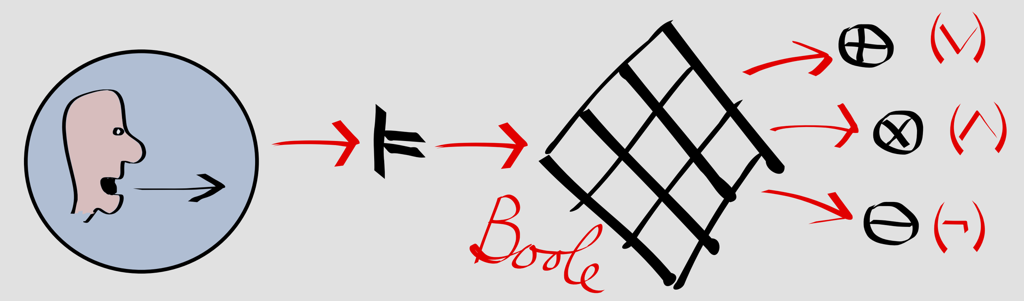


$A$ represents the information that $w_0 \in A$

A natural entailment order on such objects is:

- $A \vDash B$ iff $A \subseteq B$

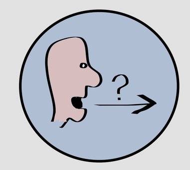
$\langle \wp\mathbf{W}, \vDash \rangle$ is a Boolean lattice, with join $\oplus$, meet $\otimes$, and complement $\ominus$ operations.




Associating these with the connectives of propositional logic gives us classical semantics.

## Comparison: meanings as requests [4]

For $\mathbf{W}$ a set of worlds, $w_0$ the actual world, we can think of a downward-closed set $A \subseteq \wp\mathbf{W}$ as:

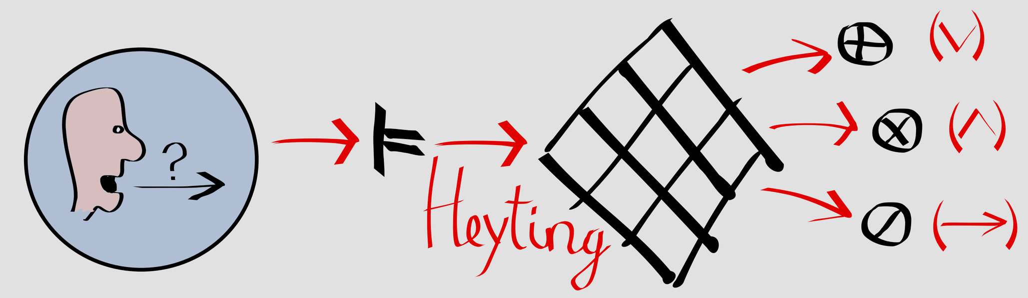


$A$ represents a request for information to establish that $w_0$ in some $\alpha \in A$

A natural entailment order on such objects is:

- $A \vDash B$ iff $\forall \alpha \in A. \exists \beta \in B. \alpha \subseteq \beta$

$\langle \wp\wp\mathbf{W}, \vDash \rangle$ is a Heyting lattice, with join $\oplus$, meet $\otimes$, and relative pseudo-complement $\oslash$.




Associating these with the logical connectives gives us basic inquisitive semantics.

## Acknowledgements

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[1]Benthem, J. van (1989). Semantic parallels in natural language and computation.
[2]Ciardelli, I., J. Groenendijk, and F. Roelofsen, (2009). Attention! Might in inquisitive semantics.
[3]Groenendijk, J., and F. Roelofsen (2009). Inquisitive semantics and pragmatics.
[4]Roelofsen, F. (2011). Algebraic foundations for inquisitive semantics.

http://staff.science.uva.nl/~westera/
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